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Little Higgs models and precision electroweak data

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Abstract: We study the low energy limit of a Little Higgs model with custodial symmetry. The method consists in eliminating the heavy fields using their classical equations of motion in the infinite mass limit. After the elimination of the heavy degrees of freedom we can directly read off deviations from the precision electroweak data. We also examine the effects on the low energy precision experiments.

Keywords: Beyond Standard Model, Higgs Physics.

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Contents

1. Introduction

One of the major problems affecting the Standard Model (SM) is the so called hierarchy problem, that is the enormous difference between the electroweak and the Planck scales. In fact, since within the SM the Higgs gets a quadratically divergent contribution to its mass, that would imply a Higgs mass of the order of the Planck mass. On the other hand LEP and SLC, together with other low energy experiments, have clearly shown that the physics behind the SM is perturbative in nature. This means that the Higgs mass cannot be very large. This requires fine tuning from the Planck scale to the electroweak scale. Clearly the situation is not satisfactory and there have been various proposals to avoid the problem. One is supersymmetry, where the quadratic divergence in the Higgs mass is cancelled by the fermions. Another one is technicolor, where the problem is solved lowering the relevant scale from the Planck mass to values of the order of TeV. Recently it has been proposed [1] to consider the Higgs fields as Nambu Goldstone Bosons (NGB) [2] of a global symmetry which is spontaneously broken at some higher scale f by an expectation value. The Higgs field gets a mass through symmetry breaking at the electroweak scale. However since it is protected by the approximate global symmetry it remains light. An important point is that the cancellation of the quadratic divergence is realized between particles of the same statistics.

Of course all models containing new physics are highly constrained by the electroweak precision tests. Aim of this paper is to consider the electroweak precision data constraints on Little Higgs models by using a general method based on the effective lagrangian approach. The idea is simple: we eliminate the heavy fields from the lagrangian via their classical equations of motion in the limit of infinite mass, which means in practice that their mass must be much bigger than m_W. We obtain an effective lagrangian in terms of the Standard model fields, from which we can directly read off the deviations.

We shall consider in detail in the following a model which exhibits an approximate SU(2) custodial symmetry. The method is quite general and can be easily applied to other models. Similar ideas are discussed in [3] for the littlest Higgs model and a class of other models. We study the electroweak precision constraints in terms of the ϵ's parameterization [4]. In order to fix our notations we shall briefly review the littlest Higgs model in section 2. Section 3 will be devoted to the study of electroweak corrections within a model which has an approximate custodial symmetry and in section 4 we will investigate the low energy precision data within both models. In appendix A we give the expressions for the couplings necessary for the evaluation of $g-2$ and in appendix B those for the weak charge.

2. Littlest Higgs model

In order to fix our notations and make contact with the existing literature we briefly consider here the littlest Higgs model. Note that our analysis in terms of the ϵ-parameters focuses only on the oblique corrections. More complete investigations of the constraints imposed by electroweak precision data on the littlest Higgs model have already been presented in the literature, using various methods [3]. We discuss it here mainly in order to show the differences with the model incorporating custodial symmetry in the following section.

The model is based on a SU(5) symmetry with a $[\mathrm{SU}(2) \times \mathrm{U}(1)]^2$ subgroup gauged. This symmetry is broken down to SO(5) by a vev of the order f. This vev also breaks the gauge symmetry to $\mathrm{SU}(2)_W \times \mathrm{U}(1)_Y$. This symmetry breaking patterns leads to 14 Goldstone bosons. Four of them are eaten up by the gauge bosons of the broken gauge group. The Goldstone boson matrix contains a Higgs doublet and a triplet under the unbroken SM gauge group. More details about this specific model and the corresponding notations can be found in ref. [3, 5].

The kinetic term for the scalar sigma model fields Σ is given by

$$\mathcal{L}_{\mathrm{kin}} = \frac{1}{2}\frac{f^2}{4}\,\mathrm{Tr}[D_\mu \Sigma D^\mu \Sigma]\,, \tag{2.1}$$

with the covariant derivative defined as

$$D_\mu \Sigma = \partial_\mu \Sigma - i(A_\mu \Sigma + \Sigma A_\mu^T)\,. \tag{2.2}$$

With A_μ we denote the gauge boson matrix:

$$A_\mu = g_1 W_\mu^{1a} Q_1^a + g_2 W_\mu^{2a} Q_2^a + g_1' B_\mu^1 Y_1 + g_2' B_\mu^2 Y_2\,, \tag{2.3}$$

where the Q_i^a are the generators of the two SU(2) groups and the Y_i are the generators of the two U(1) groups, respectively. After symmetry breaking the gauge boson matrix can be diagonalized by the following transformations:

$$
\begin{aligned}
W &= sW_1 + cW_2 \qquad & W' &= -cW_1 + sW_2 \\
B &= s'B_1 + c'B_2 \qquad & B' &= -c'B_1 + s'B_2 \,.
\end{aligned} \tag{2.4}
$$

s, c, s', and c' denote the sines and cosines of two mixing angles, respectively. They can be expressed with the help of the coupling constants:

$$
\begin{aligned}
c' &= \frac{g'}{g_2'} \qquad & s' &= \frac{g'}{g_1'} \\
c &= \frac{g}{g_2} \qquad & s &= \frac{g}{g_1}\,,
\end{aligned} \tag{2.5}
$$

with the usual SM couplings g, g', related to g_1, g_2, g_1' and g_2' by

$$
\frac{1}{g^2} = \frac{1}{g_1^2} + \frac{1}{g_2^2}\,, \qquad \frac{1}{g'^2} = \frac{1}{g_1'^{\,2}} + \frac{1}{g_2'^{\,2}}\,. \tag{2.6}
$$

The equations of motion for the heavy gauge bosons can now easily be obtained from the complete lagrangian. We neglect, at the lowest order in the momenta, derivative contributions, i.e., the contributions from the kinetic energy. Up to the order v^2/f^2 we obtain:

$$
W'^{\pm\mu} = \frac{cs}{2}(c^2 - s^2)\frac{v^2}{f^2} W^{\pm\mu} - \frac{4c^3 s}{\sqrt{2} g f^2}\left(J^{\pm\mu} - (1 - c_L) J_3^{\pm\mu}\right) \tag{2.7}
$$

$$
W'^{3\mu} = \frac{cs}{2}(c^2 - s^2)\frac{v^2}{f^2}\left(W^{3\mu} + \frac{g'}{g} B^\mu\right) - \frac{4c^3 s}{g f^2}\left(J^{0\mu} - s_L^2 \bar{t}_L \gamma^\mu t_L\right) \tag{2.8}
$$

$$
\begin{aligned}
B'^\mu &= 2c's'(c'^2 - s'^2)\frac{v^2}{f^2}\left(\frac{g}{g'} W^{3\mu} + B^\mu\right) + \\
&+ \frac{4c's'}{g' f^2}\left[(3c'^2 - 2s'^2)(J^\mu_{\rm em} + J^{0\mu}) - \frac{5}{2} c'^2 s_L^2 \bar{t}_L \gamma^\mu t_L - s_R^2 \bar{t}_R \gamma^\mu t_R\right],
\end{aligned} \tag{2.9}
$$

where we have used the notation of ref. [5] for the diagonalisation of the top sector. The currents are defined as usual and $J_3^{\pm\mu}$ describes the current of quarks of the third generation, i.e., bottom and top. Due to the mixing with the heavy top for the quarks of the third generation the neutral current is modified, too:

$$
\begin{aligned}
-Z_\mu \sqrt{g^2 + g'^2}\Bigg[& J^{0\mu}\left(1 - \frac{v^2}{f^2}\left(\frac{c^2}{2}(c^2 - s^2) + \frac{2}{5}(3c'^2 - 2s''^2)(c'^2 - s'^2)\right)\right) + \\
& + J^\mu\left(\frac{g'^2}{g^2 + g'^2} - \frac{2g}{5g'}\frac{v^2}{f^2}(3c'^2 - 2s'^2)(c'^2 - s'^2)\right) - \\
& - \frac{s_L^2}{2}\bar{t}_L \gamma^\mu t_L\left(1 - \frac{v^2}{f^2}\left(\frac{c^2}{2}(c^2 - s^2) + 2(c'^2 - s'^2)\right)\right) + \\
& + \frac{v^2}{f^2}\frac{s_R^2}{5}(c'^2 - s'^2)\bar{t}_R \gamma^\mu t_R\Bigg]\,.
\end{aligned} \tag{2.10}
$$

However this modification is irrelevant for the analysis of the precision electroweak data, as $t\bar{t}$ production at LEP was kinematically not possible. Therefore we will discard the $t\bar{t}$ correction in the following evaluations. The heavy Higgs particles completely decouple at that order.

To determine the ϵ-parameters we proceed in the same way as in ref. [6] and first look at the modification to G_F. We have two types of modifications: one directly from the mixing of the heavy W' bosons to the coupling of the charged current and the second one form the contribution of the charged current to the equations of motion of the heavy gauge bosons. The input parameters in the analysis of the electroweak data are the Fermi constant G_F, the mass of the Z vector boson m_Z and the fine-structure coupling $\alpha(m_Z)$. In terms of the model parameters we obtain:

$$\frac{G_F}{\sqrt{2}} = \frac{\alpha\pi(g^2+g'^2)}{2g^2g'^2m_Z^2}\left(1 - c^2(c^2-s^2)\frac{v^2}{f^2} + 2c^4\frac{v^2}{f^2} - \frac{5}{4}(c'^2-s'^2)^2\frac{v^2}{f^2}\right). \tag{2.11}$$

We define the Weinberg angle as [6]:

$$\frac{G_F}{\sqrt{2}} = \frac{\alpha\pi}{2s_\theta^2c_\theta^2m_Z^2}. \tag{2.12}$$

In terms of the model parameters the mass of the Z-boson is given by

$$m_Z^2 = (g^2+g'^2)\frac{v^2}{4}\left[1 - \frac{v^2}{f^2}\left(\frac{1}{6} + \frac{(c^2-s^2)^2}{4} + \frac{5}{4}(c'^2-s'^2)\right) + 8\frac{v'^2}{v^2}\right], \tag{2.13}$$

whereas the W-mass is

$$m_W^2 = \frac{g^2v^2}{4}\left[1 - \frac{v^2}{f^2}\left(\frac{1}{6} + \frac{(c^2-s^2)^2}{4}\right) + 4\frac{v'^2}{v^2}\right]. \tag{2.14}$$

The expression for the Z-mass can be used to determine the value of v for a given ratio v/f.

Our result for the corrections to the ϵ_i parameters to the order v^2/f^2 is given by:

$$\epsilon_1 = -\frac{v^2}{f^2}\left(\frac{5}{4}(c'^2-s'^2)^2 + \frac{4}{5}(c'^2-s'^2)(3c'^2-2s'^2) + 2c^4\right) + 4\frac{v'^2}{v^2} \tag{2.15}$$

$$\epsilon_2 = -2c^4\frac{v^2}{f^2} \tag{2.16}$$

$$\epsilon_3 = -\frac{v^2}{f^2}\left(\frac{1}{2}c^2(c^2-s^2) + \frac{2}{5}(c'^2-s'^2)(3c'^2-2s'^2)\frac{c_\theta^2}{s_\theta^2}\right). \tag{2.17}$$

Notice that the corrections, as they should, depend only on the parameters c, c', v/f and v'/v. Using the values of the ϵ_i parameters given in [7]

$$\begin{aligned}\epsilon_1 &= (5.1\pm1.0)\times10^{-3}\\ \epsilon_2 &= (-9.0\pm1.2)\times10^{-3}\\ \epsilon_3 &= (4.2\pm1.0)\times10^{-3}\end{aligned} \tag{2.18}$$

one can easily compare the model with data. These values only assume lepton universality and the derivation of the ϵ_i is otherwise completely model independent.

There are no stringent limits on the values of the triplet vev, such that a priori v'/v can be treated as completely arbitrary. The authors of ref. [5] obtain a bound of $v'^2/v^2 < v^2/(16f^2)$ in order to maintain a positive definite triplet mass for the Higgs. Throughout our analysis we assumed as a reasonable guideline v'/v being at least of the order v/f.

Our results are in agreement with those reported in the literature [3]. In particular for large values of v/f the allowed regions are very small, whereas for small values practically the entire parameter space is excluded. For large values of v/f this is mainly due to the fact that this model exhibits no custodial symmetry and that it is therefore difficult to satisfy the experimental constraint on ϵ_1 without fine tuning of the parameters. For small values of v/f we approach the SM limit which itself is not in agreement with the values for the ϵ-parameters. A variation of the triplet vev does in this case only slightly modify the results but does not change the general conclusions.

3. Little Higgs with custodial SU(2)

We now examine a "little Higgs" model which has an approximate custodial SU(2) symmetry [8]. The model is based on a SO(9)/[SO(5) × SO(4)] coset space, with $SU(2)_L \times SU(2)_R \times SU(2) \times U(1)$ subgroup of SO(9) gauged.

One starts with an orthogonal symmetric nine by nine matrix, representing a nonlinear sigma model field Σ which transforms under an SO(9) rotation by $\Sigma \to V\Sigma V^T$. To break the SO(4)'s to the diagonal, one can take Σ's vev to be

$$\langle\Sigma\rangle = \begin{pmatrix} 0 & 0 & \mathbb{1}_4 \\ 0 & 1 & 0 \\ \mathbb{1}_4 & 0 & 0 \end{pmatrix} \tag{3.1}$$

breaking the SO(9) global symmetry down to an SO(5)×SO(4) subgroup. This coset space has $20 = (36 - 10 - 6)$ light scalars. Of these 20 scalars, 6 will be eaten in the higgsing of the gauge groups down to $SU(2)_W \times U(1)_Y$. The remaining 14 scalars are: a single higgs doublet h, an electroweak singlet ϕ^0, and three triplets ϕ^{ab} which transform under the $SU(2)_L \times SU(2)_R$ diagonal symmetry as

$$h : (\mathbf{2}_L, \mathbf{2}_R) \qquad \phi^0 : (\mathbf{1}_L, \mathbf{1}_R) \qquad \phi^{ab} : (\mathbf{3}_L, \mathbf{3}_R)\,. \tag{3.2}$$

These fields can be written

$$\Sigma = e^{i\Pi/f}\langle\Sigma\rangle e^{i\Pi^T/f} = e^{2i\Pi/f}\langle\Sigma\rangle \tag{3.3}$$

with

$$\Pi = \frac{-i}{4}\begin{pmatrix} 0_{4\times 4} & \sqrt{2}\,\vec{h} & -\Phi \\ -\sqrt{2}\,\vec{h}^T & 0_{1\times 1} & \sqrt{2}\,\vec{h}^T \\ \Phi & -\sqrt{2}\,\vec{h} & 0_{4\times 4} \end{pmatrix} \tag{3.4}$$

where the Higgs doublet $\vec{h}$ is written as an SO(4) vector; the singlet and triplets are in the symmetric four by four matrix Φ

$$\Phi = \phi^0 + 4\phi^{ab}\, T^{l\,a} T^{r\,b}\,, \tag{3.5}$$

and the would-be Goldstone bosons that are eaten in the higgsing to $\mathrm{SU}(2)_W \times \mathrm{U}(1)_Y$ are set to zero in Π. The global symmetries protect the higgs doublet from one-loop quadratic divergent contributions to its mass. However, the singlet and triplets are not protected, and are therefore heavy, in the region of the TeV scale. The theory contains the minimal top sector with two extra coloured quark doublets and their charge conjugates. Further details and formulas can be found in [8].

The kinetic energy for the pseudo-Goldstone bosons is

$$\mathcal{L}_{kin} = \frac{f^2}{4}\,\mathrm{Tr}\left[D_\mu \Sigma D^\mu \Sigma\right] \tag{3.6}$$

and the covariant derivative is

$$D_\mu \Sigma = \partial_\mu \Sigma + i\left[A_\mu, \Sigma\right] \tag{3.7}$$

where the gauge boson matrix A_μ is defined as

$$A \equiv g_L W^{la}_{\mathrm{SO}(4)} \tau^{l\,a} + g_R W^{ra}_{\mathrm{SO}(4)} \tau^{r\,a} + g_2 W^{la} \eta^{l\,a} + g_1 W^{r3} \eta^{r\,3}\,. \tag{3.8}$$

The τ^a and η^a are the generators of two SO(4) subgroups of SO(9). For details see ref. [8].

The vector bosons can be diagonalized with the following transformations:

$$B = c' W^{r3} - s' W^{r3}_{\mathrm{SO}(4)} \qquad B' = W'^{\,r3} = s' W^{r3} + c' W^{r3}_{\mathrm{SO}(4)} \tag{3.9}$$

$$W^a = c W^{la} + s W^{la}_{\mathrm{SO}(4)} \qquad W'^a = W'^{\,la} = -s W^{la} + c W^{la}_{\mathrm{SO}(4)} \tag{3.10}$$

where the cosines and the sines of the mixing angles can be written in terms of the couplings

$$\begin{aligned} c' &= \frac{g'}{g_1} \qquad & s' &= \frac{g'}{g_R} \\ c &= \frac{g}{g_2} \qquad & s &= \frac{g}{g_L}\,. \end{aligned} \tag{3.11}$$

Again g and g' are defined in terms of g_1, g_R and g_2, g_L respectively, as in equation (2.6). Neglecting possible corrections to the currents from the top sector, we obtain the following equations of motion up to the order v^2/f^2

$$W'^{\,1,2} = -\frac{v^2\, cs}{2f^2}\left(c^2 - s^2\right) W^{1,2} + \frac{s^3 c}{f^2\, g} J^{1,2} \tag{3.12}$$

$$W'^{\,3} = -\frac{v^2\, cs}{2f^2}\left(c^2 - s^2\right)\left(W^3 - \frac{g'}{g} B\right) + \frac{s^3 c}{f^2\, g} J^3 \tag{3.13}$$

$$B' = \frac{v^2\, c' s'}{2f^2}\left(c'^{\,2} - s'^{\,2}\right)\left(\frac{g}{g'} W^3 - B\right) + \frac{s'^{\,3} c'}{f^2\, g'} J^0 \tag{3.14}$$

$$W_R^{1,2} = \frac{v^2}{2f^2} W^{1,2}\,. \tag{3.15}$$

JHEP02(2004)032

We now proceed in exactly the same way as in the previous section and look first at the modifications to G_F. The expression for G_F in terms of the model parameters is

$$\frac{G_F}{\sqrt{2}} = \frac{\alpha\pi(g^2+g'^2)^2}{2g^2g'^2}\left(1+\frac{v^2}{f^2}\frac{s^2(c^2-s^2)-s^4}{2}\right), \tag{3.16}$$

and for the neutral current we obtain

$$-Z_\mu\sqrt{g^2+g'^2}\left(J^{0\mu}\left(1+\frac{v^2}{f^2}\frac{c^2s^2-s^4+c'^2s'^2-s'^4}{2}\right)+J^\mu\left(\frac{g'^2}{g^2+g'^2}+\frac{v^2}{f^2}\frac{s'^2(c'^2-s'^2)}{2}\right)\right). \tag{3.17}$$

In this case the masses of Z- and W-bosons are given by

$$m_Z^2 = (g^2+g'^2)\frac{v^2}{4} \tag{3.18}$$

$$m_W^2 = \frac{g^2v^2}{4}\left(1+2\frac{v'^2}{v^2}\right). \tag{3.19}$$

Note that, in contrast to the littlest Higgs model discussed in section 2, the Z-mass is not modified with respect to its (tree level) standard model value and the W-mass only receives a small correction from the triplet vev. This is a consequence of the approximate custodial symmetry of the model.

The corrections to the ϵ parameters to the order v^2/f^2 are

$$\epsilon_1 = \frac{v^2}{4f^2}\left[4s'^2\left(c'^2-s'^2\right)+2c^2s^2-s^4\right]+2\frac{v'^2}{v^2} \tag{3.20}$$

$$\epsilon_2 = \frac{v^2}{4c_{2\theta}\,f^2}\Big[4s'^2\left(c'^2-s'^2\right)c_\theta^2c_{2\theta}+2s^2\left(c^2-s^2\right)\left(c_\theta^4-3c_\theta^2s_\theta^2+2c_\theta^2-s_\theta^2\right)+ \\ +s^4(c_\theta^4+s_\theta^4)\Big] \tag{3.21}$$

$$\epsilon_3 = \frac{v^2}{2s_\theta^2\,f^2}\left[s^2\left(c^2-s^2\right)\left(-c_{2\theta}+2s_\theta^2c_\theta^2\right)-s^4c_\theta^2s_\theta^2\right], \tag{3.22}$$

where we have again used the definition of s_θ and c_θ via eq. (2.12). The results of the analysis are shown in figure 1. The left panels correspond to $v'/v = v/f$ and the right panels to $v'/v = v/(32f)$, respectively. Again the allowed region lies inside the bands. Similarly to the littlest Higgs model the allowed region increases first with decreasing v/f and disappears completely upon reaching some limiting value. The latter is almost the same as in the littlest Higgs model and corresponds to the SM limit. However, in contrast to the littlest Higgs model we find reasonable agreement already for rather large values of v/f for not too large values of v'/v. This clearly shows the enhanced custodial symmetry of the model which makes it easier to satisfy the experimental constraint on ϵ_1. As can be already inferred from the expression of the W-mass, eq. (3.19), a large value of the triplet vev spoils the custodial symmetry. Therefore we find qualitative changes in the results when varying v'/v. If the triplet vev becomes too large, the constraints on ϵ_1 can no longer be satisfied easily and much more fine tuning is needed in order to remain consistent with existing experimental data.

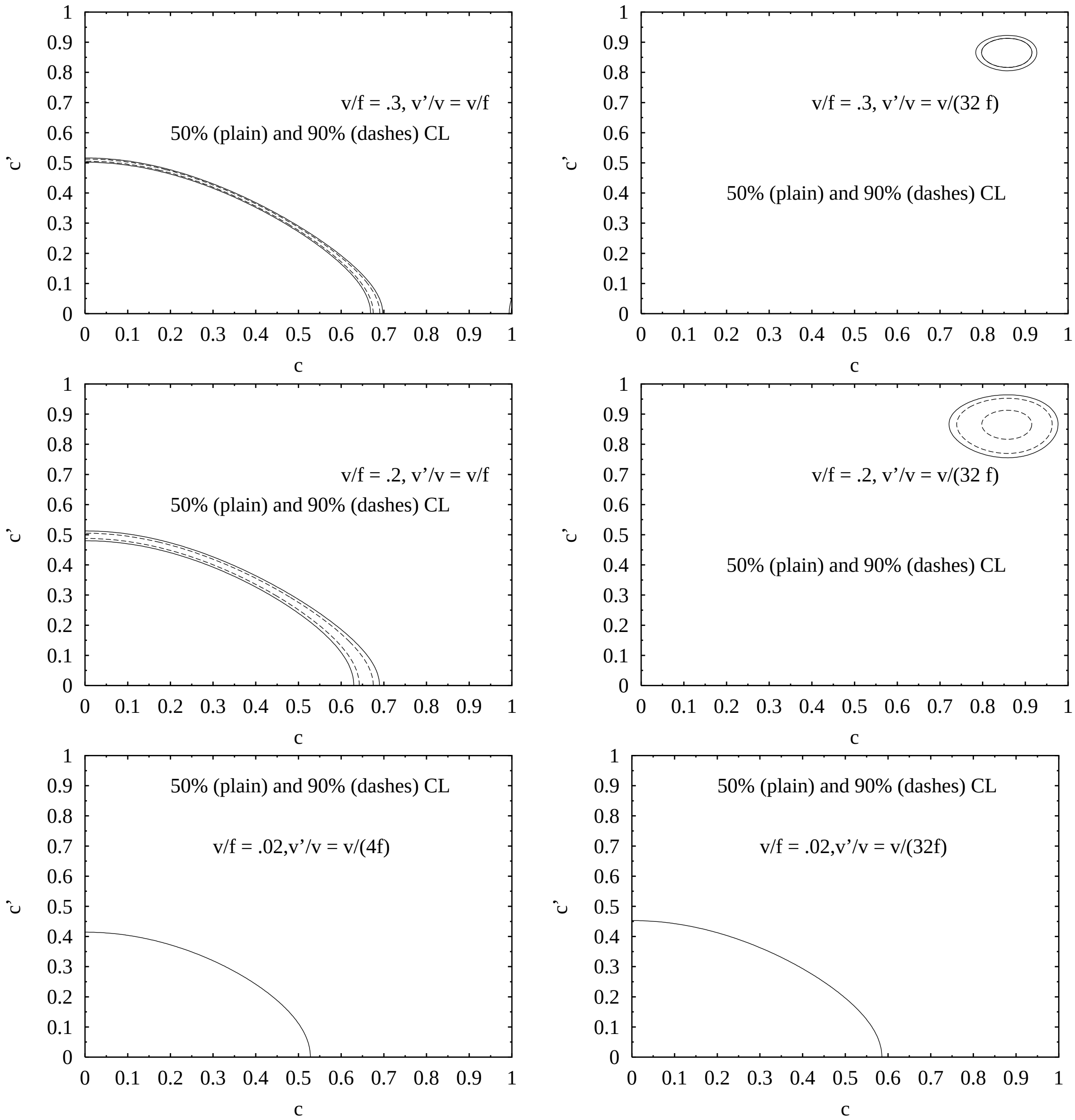

Figure 1: 90% and 50% CL exclusion contours in the plane c-c' of the cosines of the two mixing angles for four values of the ratio v/f of the vev's of the $\mathrm{SO}(9)/[\mathrm{SO}(5)\times\mathrm{SO}(4)]$ model. The allowed region lies inside the 90% and 50% bands, respectively.

4. Low energy precision data

Precision experiments at low energy allow a precise determination of the $g-2$ of the muon and of the weak charge of cesium atoms. We will analyse these data in order to see whether they can put constraints on our models. In a first step we will examine $g-2$ within the littlest Higgs model. To that end we will use a somewhat different technique than in the previous sections. Instead of deriving an effective lagrangian by integrating out the heavy degrees of freedom we will use the linearized version of the model as presented in ref. [5] and explicitly include corrections from the heavy bosons.

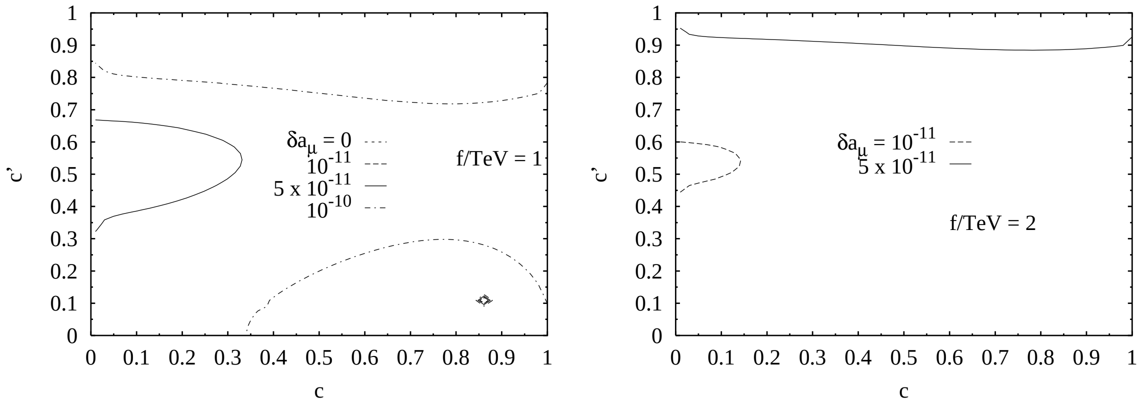

Figure 2: Corrections to $g-2$ of the muon as a function of c, c' for $v'^2/v^2 = v^2/(24f^2)$.

4.1 $g-2$ of the muon in the littlest Higgs model

We can use the results of ref. [9] to calculate the corrections to $g-2$ of the muon which we will denote by a_μ. The relevant contributions are discussed in appendix A.

The difference between experiment and the standard model prediction for a_μ is [11]

$$\delta a_\mu = a_\mu^{\rm exp} - a_\mu^{\rm SM} = 17(18) \times 10^{(-10)}\,. \tag{4.1}$$

The numerical results within the littlest Higgs model are relatively insensitive to the choice of parameter values of the model. We obtain a difference from the standard model value of at most $\delta a_\mu = a_\mu^{\rm LH} - a_\mu^{\rm SM}$ of the order of 1×10^{-10}. The contributions of the additional heavy particles are thereby completely negligible and the dominant contributions arise from the corrections to the light Z and W couplings. Thus the analysis of ref. [10] is not complete. In figure 2 we display δa_μ for two different values of the symmetry breaking scale f as a function of the cosines of the mixing angles c, c'. For larger values of f the corrections become even smaller. We thereby took the Higgs mass to be 113 GeV and used as experimental input m_Z, α, and G_F as before. A variation of the triplet vev does not change these findings. The results in the model with custodial symmetry are in general closer to the SM limit than in the littlest Higgs model. Thus we expect even smaller corrections in that case and we shall not give an explicit evaluation of $g-2$ within the model with approximate custodial symmetry.

4.2 Weak charge of cesium atoms

At low energy, parity violation in atoms is due to the electron-quark effective lagrangian

$$\mathcal{L}_{\rm eff} = \frac{G_F}{\sqrt{2}} (\bar{e}\gamma_\mu\gamma_5 e)(C_{1u}\bar{u}\gamma^\mu u + C_{1d}\bar{d}\gamma^\mu d)\,. \tag{4.2}$$

The experimentally measured quantity is the so-called "weak charge" defined as

$$Q_W = -2\left(C_{1u}(2Z+N) + C_{1d}(Z+2N)\right), \tag{4.3}$$

where Z, N are the number of protons and neutrons of the atom, respectively.

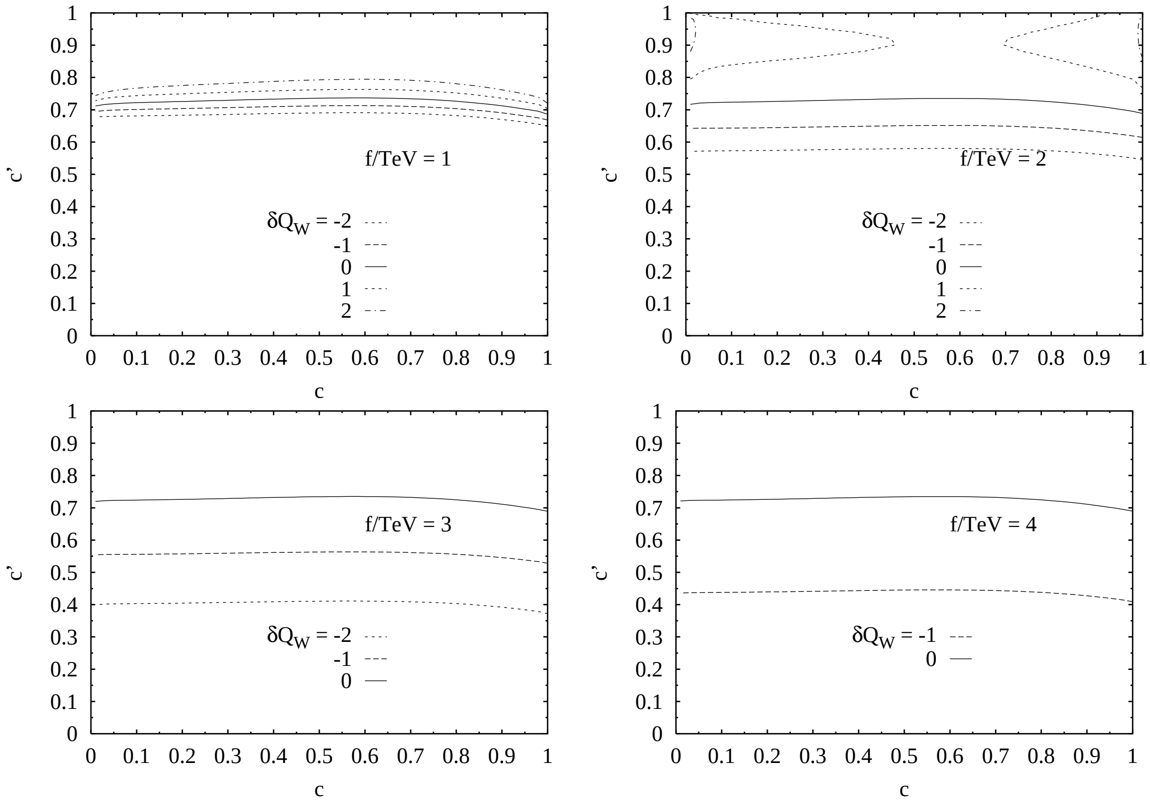

Figure 3: Corrections to the weak charge of cesium atoms as a function of c and c' in the littlest Higgs model.

The effective lagrangian, eq. (4.2), can be derived from the interaction of Z, Z_H, and A_H with the fermions by integrating out the heavy degrees of freedom. The corresponding expressions for the littlest Higgs model as well as for the model with approximate custodial symmetry are given in appendix C.

Recently precise data on cesium atoms have been reported in ref. [12]:

$$Q_W(Cs)^{\mathrm{exp}} = -72.2 \pm 0.8 \,. \tag{4.4}$$

The standard model prediction is [13]

$$Q_W(Cs)^{\mathrm{SM}} = -73.19 \pm 0.13 \,. \tag{4.5}$$

Thus

$$\delta Q_W(Cs) = Q_W(Cs)^{\mathrm{exp}} - Q_W^{\mathrm{SM}} = 0.99 \pm 0.93 \,. \tag{4.6}$$

The difference of the weak charge of Cs in the littlest Higgs model and the standard model is shown in figure 3 for different values of f in the littlest Higgs model and in figure 4 for the model with approximate custodial symmetry. As experimental input for our analysis we have again used m_Z, G_F, and α. In order to discuss the weak charge result, let's consider the value $\delta Q_W(Cs) = 1$ which is close to the present experimental central value. It is clear from figure 3 and 4 that the value of the high scale f should be in the range of few

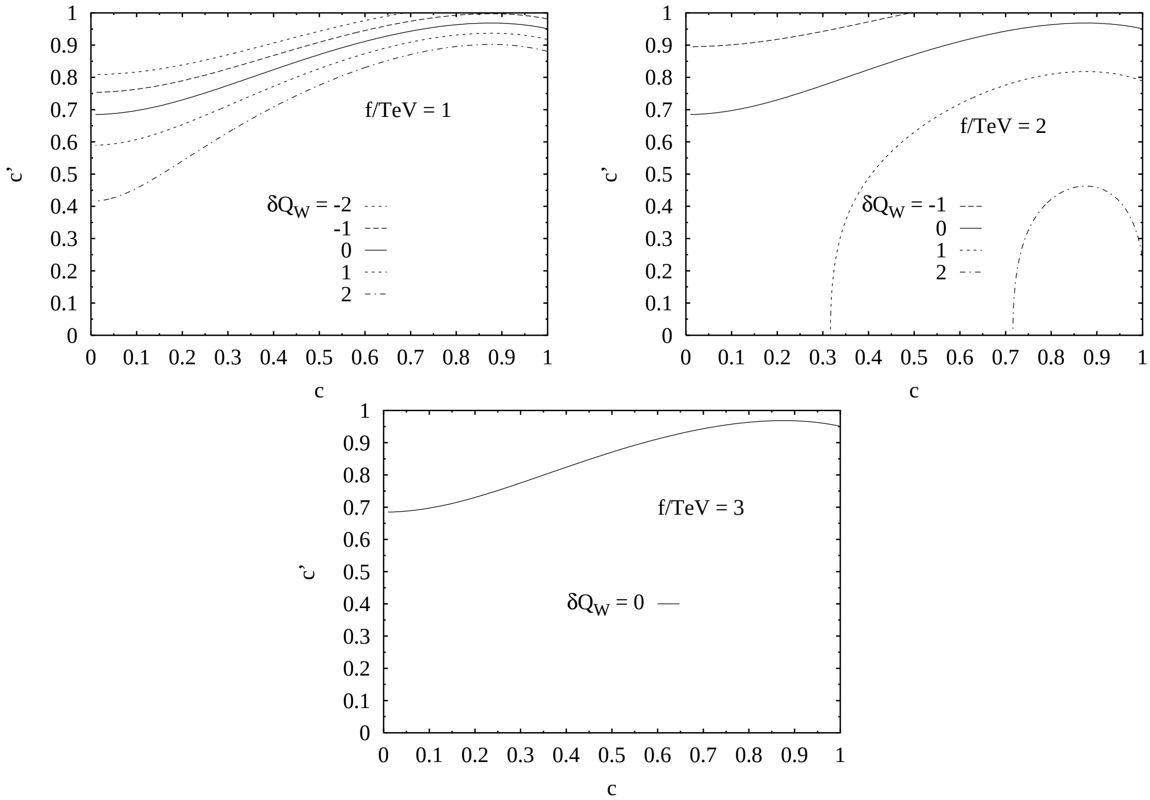

Figure 4: Corrections to the weak charge of cesium atoms as a function of c and c' in the little Higgs model with approximate custodial symmetry.

TeV in order to obtain the measured deviation. The allowed scale is slightly lower in the custodial model with respect to the non-custodial one as the custodial model is closer to the standard model in its predictions. When the scale f is too large the new physics effects become negligible. The scale f in the few TeV range is consistent with what is expected on the model-building side and from the LEP data for little Higgs model. Obviously this result should be taken only as a first indication as the error on $\delta Q_W(Cs)$ is large.

5. Conclusions

In this paper we have studied the low energy limit of a Little Higgs model incorporating an approximate custodial symmetry. For illustrational purposes we briefly presented in section 2 the corresponding analysis for the "littlest Higgs" model which has no custodial symmetry. In order to study the constraints coming from the LEP/SLC experiments we have used a method which consists in eliminating the heavy degrees of freedom. We find, in agreement with earlier studies in the literature [3, 5], rather stringent limits on the littlest Higgs model imposed by existing electroweak precision data. This is mainly due to the difficulty of the model to accommodate for the experimental results of the ρ parameter. Our main focus lied on the study of a model where custodial symmetry is approximately fulfilled. As long as the value of the Higgs triplet vev, which violates custodial symmetry, does not become too large, we need much less fine tuning than in the littlest Higgs model in order to satisfy the experimental constraints. Thus custodial symmetry seems to be an essential ingredient for little Higgs models.

In the second part of this paper we look at the constraints from low energy precision data., i.e., $g-2$ of the muon and to the atomic "weak charge" of the cesium. To that end we apply a slightly different method: To evaluate the corrections to these quantities the contributions of the heavy degrees of freedom have directly been taken into account. The analysis of the low energy precision data does not change the above conclusions. For $g-2$ of the muon the corrections are simply too small to impose any new constraints on the model parameters. The actual state of precision for the weak charge does not allow for establishing new constraints either, even if the corrections are not negligible.

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A. Details for the calculation of $g-2$ of the muon in the Littlest Higgs model

The relevant one-loop Feynman diagrams are shown in figure 5. For graph 5a we have contributions from the exchange of a light and a heavy Z and a light and a heavy photon. Since we have no flavour mixing interaction the fermion in the intermediate state can only be a muon. The contribution of the photon is not modified with respect to its standard model value (note that $\mathrm{U}(1)_{\mathrm{em}}$ is not broken so that this should be the case).

Explicitly we have

$$
\begin{aligned}
[a_\mu]_a = \frac{1}{8\pi^2}\Bigg\{ & C_V^2\left[\frac{z}{3} + z^2\left(\frac{25}{12} + \ln(z)\right) + z^3\left(\frac{97}{10} + 6\ln(z)\right) + z^4\left(\frac{208}{5} + 28\ln(z)\right)\right] + \\
& + C_A^2\left[-5\frac{z}{3} - z^2\left(\frac{19}{12} + \ln(z)\right) - z^3\left(\frac{77}{15} + 4\ln(z)\right) - z^4\left(\frac{173}{10} + 14\ln(z)\right)\right]\Bigg\} + \\
& + \mathcal{O}(z^5)\,, \qquad \text{(A.1)}
\end{aligned}
$$

where $z = m_\mu^2/M_g^2$. With M_g we denote the mass of the exchanged gauge boson, i.e., m_Z, M_{Z_H} or M_{A_H}. C_V and C_A can be extracted from the vector and axial couplings of the corresponding gauge bosons to the muons, see ref. [9]. The corresponding expressions are listed in appendix B.

For graph 5b we obtain contributions from light and heavy charged W bosons. The intermediate fermion has to be neutral, i.e., it is a neutrino. If we neglect the mass of the neutrino we obtain

$$
[a_\mu]_b = \frac{C_V^2}{4\pi^2}\left(5\frac{z}{3} + \frac{z^2}{3} + 3\frac{z^3}{20} + \frac{z^4}{12}\right) + \mathcal{O}(z^5)\,, \qquad \text{(A.2)}
$$

where we note again $z = m_\mu^2/M_g^2$. M_g can be the mass of the light or the heavy W bosons. We also used the fact that $C_V = -C_A$.

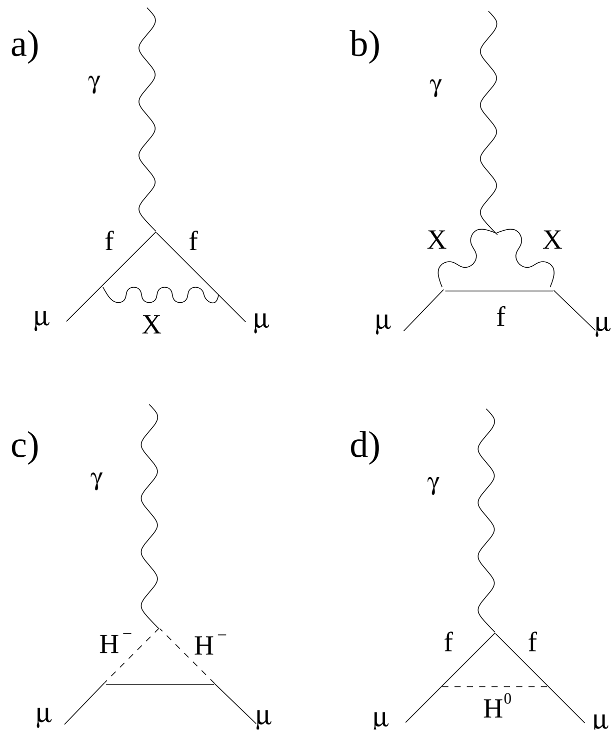

Figure 5: Loop graphs contributing to the weak correction to Δg. (a) and (b) correspond to the exchange of a vector boson X while (c) and (d) are the Higgs sector contributions.

Graph 5c receives contributions from singly charged scalars and a neutrino in the intermediate states. There are no contributions from doubly charged scalars since they do not couple to the corresponding fermions. Neglecting again the neutrino mass the result can be written

$$[a_\mu]_c = \frac{C_S^2}{8\pi^2}\left(\frac{z}{6}+\frac{z^2}{12}+\frac{z^3}{20}+\frac{z^4}{30}\right)+\mathcal{O}(z^5)\,, \tag{A.3}$$

with $z = m_\mu^2/M_H^2$. The pseudoscalar coupling (C_P) vanishes.

For graph 5d we have to consider muons and a neutral scalar as intermediate states. The result is

$$\begin{aligned}[a_\mu]_d = \frac{1}{8\pi^2}\Bigg\{ & -C_S^2\Bigg[z\left(\frac{7}{6}+\ln(z)\right)+z^2\left(\frac{39}{12}+3\ln(z)\right)+\\ & \qquad +z^3\left(\frac{201}{20}+9\ln(z)\right)+z^4\left(\frac{484}{15}+28\ln(z)\right)\Bigg]+\\ & +C_P^2\Bigg[z\left(\frac{11}{6}+\ln(z)\right)+z^2\left(\frac{89}{12}+5\ln(z)\right)+\\ & \qquad +z^3\left(\frac{589}{20}+12\ln(z)\right)+z^4\left(\frac{1732}{15}+84\ln(z)\right)\Bigg]\Bigg\}+\mathcal{O}(z^5)\,,\end{aligned} \tag{A.4}$$

where $z = m_\mu^2/M_h^2$, with M_h being the mass of the neutral scalar.

B. Couplings for the calculation of $g-2$ in the littlest Higgs model

The vector and axial vector couplings are given as follows. For the Z we obtain up to the order $\mathcal{O}(v^2/f^2)$:

$$C_V = \frac{g}{2c_\theta}\left[2s_\theta^2 - \frac{1}{2} + \frac{v^2}{f^2}\left(3s_\theta \frac{x_Z^B}{s'c'}\left(\frac{c'^2}{2} - \frac{1}{5}\right) - c_\theta x_Z^W \frac{c}{2s}\right)\right] \tag{B.1}$$

$$C_A = \frac{g}{2c_\theta}\left[\frac{1}{2} + \frac{v^2}{f^2}\left(s_\theta \frac{x_Z^B}{s'c'}\left(\frac{c'^2}{2} - \frac{1}{5}\right) + c_\theta x_Z^W \frac{c}{2s}\right)\right]. \tag{B.2}$$

For the heavy Z we obtain:

$$C_V = C_A = \frac{gc}{4s}. \tag{B.3}$$

Note that it is sufficient to retain the leading order contributions for the couplings of the heavy bosons since their contributions to $g-2$ are already suppressed by one order in v^2/f^2 due to the mass. The heavy photon couplings are given by

$$C_V = 3\frac{g'}{2s'c'}\left(\frac{c'^2}{2} - \frac{1}{5}\right) \tag{B.4}$$

$$C_A = \frac{g'}{2s'c'}\left(\frac{c'^2}{2} - \frac{1}{5}\right). \tag{B.5}$$

For the W bosons we obtain:

$$C_V = -C_A = \frac{g}{2\sqrt{2}}\left(1 - \frac{v^2}{f^2}\frac{c^2}{2}(c^2 - s^2)\right), \tag{B.6}$$

and for the heavy W-bosons:

$$C_V = -C_A = \frac{gc}{2\sqrt{2}s}. \tag{B.7}$$

The relevant Higgs couplings are given by

$$C_S = \frac{m_\mu}{\sqrt{2}v}\left(\frac{v}{f} - 4\frac{v'}{v}\right) \tag{B.8}$$

$$C_P = 0 \tag{B.9}$$

for the heavy Higgs and

$$C_S = \frac{m_\mu}{v}\left(1 - 4\frac{v'^2}{v^2} - \frac{2}{3}\frac{v^2}{f^2}\right) \tag{B.10}$$

$$C_P = 0. \tag{B.11}$$

for the light higgs which contributes only to graph $5d$ (see figure 3). Note that there are no leading order contributions to the couplings of the heavy Higgs, i.e., at the order we consider the heavy Higgs contribution vanishes.

JHEP02(2004)032

C. Parameters of the effective lagrangian for the weak charge

In the littlest Higgs model we obtain

$$C_{1u}^{A_H} = -\frac{\sqrt{2}}{M_{A_H}^2 G_F}\frac{\alpha\pi}{c_\theta^2 s'^2 c'^2}\left(-\frac{1}{5}+\frac{c'^2}{2}\right)\left(\frac{1}{3}-\frac{5}{6}c'^2\right) \tag{C.1}$$

$$C_{1d}^{A_H} = -\frac{\sqrt{2}}{M_{A_H}^2 G_F}\frac{\alpha\pi}{c_\theta^2 s'^2 c'^2}\left(-\frac{1}{5}+\frac{c'^2}{2}\right)\left(-\frac{1}{15}+\frac{1}{6}c'^2\right) \tag{C.2}$$

$$C_{1u}^{Z_H} = -C_{1d}^{Z_H} = -\frac{\sqrt{2}}{M_{Z_H}^2 G_F}\frac{\alpha\pi c^2}{4 s_\theta^2 s^2} \tag{C.3}$$

$$\begin{aligned} C_{1u}^{Z} = & -\frac{\sqrt{2}}{m_Z^2 G_F}\frac{\alpha\pi}{s_\theta^2 c_\theta^2}\left\{\frac{1}{4}-\frac{2}{3}s_\theta^2+\frac{v^2}{f^2}\left[\left(\frac{1}{2}-\frac{4}{3}s_\theta^2\right)\left(c_\theta x_Z^{W'}\frac{c}{2s}+\frac{s_\theta x_Z^{B'}}{s'c'}\left(-\frac{1}{5}+\frac{c'^2}{2}\right)\right)+\right.\right. \\ & \left.\left. +\frac{1}{2}\left(c_\theta x_Z^{W'}\frac{c}{2s}+\frac{s_\theta x_Z^{B'}}{s'c'}\left(-\frac{1}{3}-\frac{c'^2}{6}\right)\right)\right]\right\} \end{aligned} \tag{C.4}$$

$$\begin{aligned} C_{1d}^{Z} = & -\frac{\sqrt{2}}{m_Z^2 G_F}\frac{\alpha\pi}{s_\theta^2 c_\theta^2}\left\{-\frac{1}{4}+\frac{1}{3}s_\theta^2+\frac{v^2}{f^2}\left[\left(-\frac{1}{2}+\frac{2}{3}s_\theta^2\right)\left(c_\theta x_Z^{W'}\frac{c}{2s}+\frac{s_\theta x_Z^{B'}}{s'c'}\left(-\frac{1}{5}+\frac{c'^2}{2}\right)\right)+\right.\right. \\ & \left.\left. +\frac{1}{2}\left(-c_\theta x_Z^{W'}\frac{c}{2s}+\frac{s_\theta x_Z^{B'}}{s'c'}\left(-\frac{1}{15}+\frac{c'^2}{6}\right)\right)\right]\right\}, \end{aligned} \tag{C.5}$$

with

$$x_Z^{W'} = -s\,c\,\frac{c^2-s^2}{2c_\theta} \tag{C.6}$$

$$x_Z^{B'} = -5\,c'\,s'\,\frac{c'^2-s'^2}{2s_\theta}\,. \tag{C.7}$$

In the model with approximate custodial symmetry we get:

$$C_{1u}^{A_H} = \frac{\sqrt{2}}{M_{A_H}^2 G_F}\frac{\alpha\pi}{3c_\theta^2 c'^2}\left(1+\frac{s'^2}{4}\right) \tag{C.8}$$

$$C_{1d}^{A_H} = -\frac{\sqrt{2}}{M_{A_H}^2 G_F}\frac{\alpha\pi}{6c_\theta^2 c'^2}\left(1-\frac{s'^2}{2}\right) \tag{C.9}$$

$$C_{1u}^{Z_H} = -C_{1d}^{Z_H} = -\frac{\sqrt{2}}{M_{Z_H}^2 G_F}\frac{\alpha\pi s^2}{4 s_\theta^2 c^2} \tag{C.10}$$

$$\begin{aligned} C_{1u}^{Z} = & -\frac{\sqrt{2}}{m_Z^2 G_F}\frac{\alpha\pi}{s_\theta^2 c_\theta^2}\left[\frac{1}{4}-\frac{2}{3}s_\theta^2+\frac{v^2}{24f^2}\Big(-2(c'^2-s'^2)+s'^2(c'^2-s'^2)(1-4s_\theta^2)-\right. \\ & \left. -s^2(c^2-s^2)(1-4c_\theta^2)\Big)\right] \end{aligned} \tag{C.11}$$

$$\begin{aligned} C_{1d}^{Z} = & -\frac{\sqrt{2}}{m_Z^2 G_F}\frac{\alpha\pi}{s_\theta^2 c_\theta^2}\left[\frac{1}{3}s_\theta^2-\frac{1}{4}+\frac{v^2}{24f^2}\Big(-s^2(c^2-s^2)(1+2c_\theta^2)+\right. \\ & \left. +(c'^2-s'^2)(1-2c_\theta^2 s'^2)\Big)\right]. \end{aligned} \tag{C.12}$$

Note that the expressions do not depend on the triplet vev v'.

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JHEP02(2004)032

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JHEP02(2004)032

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Erratum

In formula (3.22) for the parameter ϵ_3 in the custodial model the last addendum is missing. The correct formula reads

$$\epsilon_3 = \frac{v^2}{2s_\theta^2 f^2}\left[s^2\left(c^2 - s^2\right)\left(-c_{2\theta} + 2s_\theta^2 c_\theta^2\right) - s^4 c_\theta^2 s_\theta^2\right] - \frac{2c_\theta^2}{s_\theta^2}\frac{v'^2}{v^2}\,.$$

The numerical analysis of our paper is unchanged as the extra addendum was correctly taken into account.

We also remark that our formulas for the custodial model imply only the presence of a vev for the $Y = 0$ triplet Higgs. If one wishes to include also a vev v'' for the $Y = 1$ triplet Higgs, one simply has to apply the following substitution in the ϵ parameter formulas:

$$v'^2 \to v'^2 - v''^2\,.$$

In this case a large value for the difference of the squares of the triplet vevs spoils the custodial symmetry, however one can argue that the two different triplet vevs should be similar in size and at least partially compensate their effects. A precise evaluation of this effect is not possible in the effective theory since there are unknown order one factors in the radiatively generated potential.

JHEP02(2004)032